

A MATHEMATICAL MODEL FOR DECOMPRESSION

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Abstract—A mathematical model for decompression is presented. It includes a first-order linear differential equation, which models the diffusion of gases within the body, as well as initial conditions and a nonlinear constraint inequality which insures safety from decompression sickness. The model is solved (numerically and/or analytically) under various hypotheses, and the results are compared with the U.S. Navy Dive Table.

INTRODUCTION

Occasionally, human beings will enter an environment different from their normal one, in which changes in pressure on the body may result in a buildup of air bubbles in the tissues. This condition is known as decompression sickness or, more commonly, the “bends.” Scuba diving is a popular sport in which participants subject themselves to changes in pressure. Other examples are provided by the man-in-space program and certain tunnelling operations in which air pressure on the body is alternately increased and decreased.

During respiration, air is inspired into the lungs, from which it enters the blood. The air in solution in the blood is then carried to the body tissues where it is released by the blood into the tissues. The driving force for the exchange of gas at either the lungs or the body tissues is the difference in the partial pressures of the different gases on either side of the transaction site.

In the normal situation in which the pressure on the body is decreased, the pressure of the air that is breathed is less than that in solution in the body tissues. When this happens, the body will act to relieve the difference in pressure. The blood will pick up excess gas in solution in the tissues and transfer it to the lungs from which it is exhaled. A dangerous situation occurs when the outside pressure is decreased so much that the tissue–blood–lung transfer of excess gas cannot take place rapidly enough. In this case, the gas pressure in the tissues will force the excess gas out of solution, resulting in the formation of gas bubbles in the tissues.

The diffusion of a gas between lungs and blood and between blood and tissues can be modeled mathematically. Henry’s Law is a linearized pressure law which guarantees that a gas will flow from a higher pressure region to a lower pressure region at a rate which is proportional to the difference in the partial pressure of the gas in the two different regions.

In what follows, we shall be concerned mainly with the effect of nitrogen and carbon

dioxide in the body tissues. Nitrogen comprises approximately 79% of the air we normally breath (including scuba divers who breath compressed air), and it doesn't react very much with elements in the body tissues.

On the other hand, oxygen, the other major component (20%) of the air we breath, does interact with other elements in the body, and it is consumed during these intratissue chemical reactions. Thus, when the external pressure is reduced, the difference in partial pressure of oxygen between the tissues and the external environment will increase at a rate less than that of nitrogen. Therefore, all other things being equal, oxygen is much less likely than nitrogen to be the cause of decompression sickness.

The other gas which may lead to the onset of decompression sickness is carbon dioxide. It forms a negligible fraction (less than 0.5%) of the air we inhale. However, it is a product of the oxidation of elements in the tissues, and the air we exhale normally contains about four percent carbon dioxide. This production of carbon dioxide will lead to higher partial pressure differences, which, in turn, may lead to the formation of gas bubbles in the tissues.

Other gases form such a minute fraction of the air we breath that we shall ignore them. Thus, we can single out carbon dioxide and nitrogen as the two gases which initially cause the onset of decompression sickness.

The first systematic study of decompression was done by Haldane [1, 2] in the early part of the twentieth century. He conducted experiments, mainly on goats, and concluded that the body could withstand a reduction in outside air pressure by approximately 50% without suffering from decompression sickness. More recently, this theory has been refined.

Concern was first focused on the problem during World War II when U.S. Navy frogmen were called upon to perform military operations underwater. Since that time, the U.S. Navy and other agencies have performed numerous experiments and collected a large mass of data, which led to the compilation of the U.S. Navy Standard Air Decompression Table [3] which now serves as a standard guide for scuba divers. During that time, the mathematical theory of decompression has also evolved to its present form. See, for example, [1-9].

In the remainder of this paper, we present the basic model and show its solution under various hypotheses. We start with the Haldane model and proceed to more refined models. It should be noted that the model we present [see Eq. (2)] assumes that diffusion takes place instantaneously from the body tissues to the external environment. In actuality, there is a delay due to the time it takes for the blood to transfer gas from the tissues to the lungs. However, if the external pressure does not change too rapidly, there will be a smooth flow of gas from tissues to blood to lungs, and the model should be adequate.

THE MODEL

Let us consider the case of a scuba diver, diving in the ocean, breathing compressed air. At sea level, atmospheric pressure is approximately 14.7 pounds per square inch (psi). Seawater weighs approximately 64 pounds per cubic foot. An elementary calculation shows that the pressure at the bottom of a 33 foot high column of seawater will be double that at the top. In other words, the pressure will increase by one atmosphere (14.7 psi) for each 33 feet of descent below the surface of the ocean (assuming seawater is incompressible). Because of this equivalence (1 atmosphere = 33 feet seawater = 14.7 psi), we choose to measure pressure using units of feet-seawater (fsw), so that depth and pressure can be equated.

We introduce the following notation to be used in our model:

$$P = P(t) = \text{ambient (external) pressure,}$$

$$A = A(t) = \text{pressure of gas in tissues,}$$

$$d = d(t) = \text{depth below the surface,}$$

$$D = D(t) = \text{absolute depth} = d + 33.$$

Here, t represents time. Since ambient pressure is proportional to absolute depth, we let

$$P = k_1 D,$$

and we can choose $k_1 = 1$ by choosing the units so that both P and D are measured in fsw. Another quantity which we shall be interested in is the rate of ascent, $r(t)$, which satisfies the equation

$$r(t) = -\frac{dD}{dt} = -\frac{dP}{dt}. \quad (1)$$

We are interested in finding the optimum safe rate of ascent (in fsw/min)—the rate which will allow the diver to ascend to the surface in the least time and which will also ensure that the diver does not develop decompression sickness. This optimum ascent rate is chosen so that the ratio of the partial pressure of gas in the tissues to that in the lungs (A/P) never exceeds a predetermined value, $f(A)$, which may depend on the tissue pressure.

In order to model the situation mathematically, recall that the rate of exchange of gas between the lungs and blood and between the blood and tissues is proportional to the difference between the partial pressure of the gas in the blood and the partial pressure of the gas in the tissues or in the lungs. Therefore, we must solve the differential equation

$$\frac{dA}{dt} = k_2(P - A), \quad (2)$$

with the initial condition

$$A(0) = D(0) = A_0. \quad (3)$$

Here, k_2 is the constant of proportionality, and it depends on the tissue. It will be positive, since tissue pressure increases (decreases) if $P > A$ ($P < A$). If τ represents tissue half-time (time during which the difference between tissue pressure and ambient pressure is halved), then

$$k_2 = \ln 2 / \tau. \quad (4)$$

In solving the model, we must also impose the condition that $A/P \leq f(A)$.

SOLUTION OF THE MODEL

The solution of the initial value problem (2)–(3) for A is

$$A(t) = A_0 e^{-k_2 t} + k_2 \int_0^t e^{-k_2(t-s)} P(s) ds. \quad (5)$$

In order to solve for A and P , we use the condition that

$$A(t) \leq f(A(t)) \cdot P(t). \quad (6)$$

The function f can be experimentally determined and tabulated. Of course, it depends not only on A but also on the properties of the tissue. We solve (5) and (6) for A and P , and determine the optimal ascent rate from (1).

THE HALDANE MODEL

The earliest and simplest theory of decompression, proposed by Haldane [1, 2], states that ambient pressure could safely be halved. This implies that $f(A) \equiv 2$. The problem then becomes one of solving the inequality

$$A_0 e^{-k_2 t} + k_2 \int_0^t e^{-k_2(t-s)} P(s) ds \leq 2P(t). \quad (7)$$

The optimal (least time) solution of this problem would require that $A/P = 2$ for all $t > 0$. Otherwise the difference in partial pressure would not be large enough to allow the elimination of the excess air in the least time. Thus, the optimal solution of this mathematical problem requires the diver to ascend immediately from depth A_0 to depth $A_0/2$, and to continue to ascent from that point to the surface according to the exponential solution

$$P(t) = (A_0/2) e^{-k_2 t/2}$$

of

$$A_0 e^{-k_2 t} + k_2 \int_0^t e^{-k_2(t-s)} P(s) ds = 2P(t), \quad (7a)$$

which is (7) with the inequality replaced by equality.

The quick initial ascent is impractical for two reasons. First, it is impossible to ascend at an infinite rate. Second, even if the diver were to ascend at a very high but finite rate to depth $A_0/2$, he would expose himself to a potentially far more serious danger, that of an air embolism (a problem we shall not pursue further here). Therefore, we shall assume that the diver makes his ascent at a rate of 60 fsw/min. There are several reasons for using this figure. First, as long as the diver is careful, he should not be in danger of an embolism at this rate. Second, the diver can monitor his ascent rate by following his air bubbles as they rise, since air bubbles will rise at approximately 60 fsw/min. Third, the calculations become easier if we assume that the diver ascends at the rate of 1 fsw/sec.

The problem thus is one of solving (7) for $P(t)$. We get

$$P(t) = \begin{cases} -60t + A_0, & 0 \leq t \leq t_1, \\ P_1(t), & t_1 \leq t \leq t_2. \end{cases} \quad (8)$$

Here, t_1 is the smallest positive solution of

$$A(t_1) = 2P(t_1). \quad (9)$$

$P_1(t)$ is found by solving

$$A(t) = 2P_1(t), \quad t_1 \leq t \leq t_2, \quad (10)$$

and t_2 is the total ascent time.

EXAMPLE USING THE HALDANE MODEL

Suppose that a diver is at a depth of 99 feet below the surface ($D = 132$), and is saturated with gas at that depth. The Haldane model says that he could safely ascend immediately to

a depth of 33 feet ($D = 66$). To complete the solution to the problem, we make the additional assumption that all tissue half-times are the same, say 20 min. This assumption is unreasonable for obvious reasons, but it does simplify the calculations and discussion. Later, this assumption will be modified. We then find, from (4), that $k_2 = 0.0346$. Substituting (8) into (9), we find that t_1 must solve

$$132 e^{-k_2 t_1} + k_2 \int_0^{t_1} e^{-k_2(t_1-s)}(-60s + 132) ds = 2(-60t_1 + 132).$$

This reduces to

$$e^{-k_2 t_1} - k_2 t_1 = 1 - \frac{11}{5} k_2,$$

whose solution is

$$t_1 = 1.11 \text{ min} = 66.7 \text{ sec.}$$

Thus, the diver would ascend at the constant rate of 60 fsw/min for 66.7 sec, until he reached a depth of 32.3 feet ($D = 65.3$). (Note: he could rise to a depth, d , slightly less than 33 feet, since he was exhaling some excess gas during the 66.7 sec ascent, thus relieving some of the excess pressure.) Now, for $t > t_1$, we must solve (10):

$$132 e^{-k_2 t} + k_2 \int_0^{t_1} e^{-k_2(t-s)}(-60s + 132) ds + k_2 \int_{t_1}^t e^{-k_2(t-s)} P_1(s) ds = 2P_1(t). \quad (11)$$

Equation (11) is a Volterra integral equation of the second kind for $P_1(t)$. One way to solve this equation is by the method of resolvent kernels. An easier way is to reduce it to a differential equation. We have

$$P_1(t) = C_1 e^{-k_2 t} + \frac{k_2}{2} \int_{t_1}^t e^{-k_2 t} e^{k_2 s} P_1(s) ds.$$

Multiplying by $e^{k_2 t}$, we obtain

$$P_1(t) e^{k_2 t} = C_1 + \frac{k_2}{2} \int_{t_1}^t e^{k_2 s} P_1(s) ds.$$

We let

$$y(t) = P_1(t) e^{k_2 t},$$

so that

$$y(t) = C_1 + \frac{k_2}{2} \int_{t_1}^t y(s) ds,$$

which, upon differentiation, yields

$$y' = \frac{k_2}{2} y.$$

The solution for P is

$$P_1(t) = C_2 e^{(-k_2/2)t}. \quad (12)$$

C_2 can be evaluated using

$$P(t_1) = P_1(t_1) = C_2 e^{(-k_2/2)t_1} = 65.3,$$

so that

$$C_2 = 65.3 e^{(k_2/2)t_1} = 66.6.$$

We now evaluate t_2 from

$$P(t_2) = C_2 e^{(-k_2/2)t_2} = 33,$$

which yields

$$t_2 = 40.5 \text{ min.}$$

Thus, if the diver follows the optimum ascent rate (with the specified restrictions) given by (1) and (8), he would surface 40.5 min after he left the bottom, without suffering the effects of the "bends," *if we are to believe this model*. Note: the U.S. Navy table requires more than three times that amount of time for ascent.

We are assuming that the diver can monitor his rate of ascent so that he ascends at an exponential rate (anaphrotropic decompression [4]). It is unreasonable to expect the diver to be able to do this. More commonly, the diver will ascend at a constant rate (60 fsw/min) until he reaches a certain depth, at which he stops (first decompression stop) for a specified period of time. Then he will ascend to a second decompression stop. This process continues until he is able to surface without letting $A(t)$ become larger than $2P(t)$. In this case,

$$P(t) = \begin{cases} -60t + A(0), & 0 \leq t \leq t_3 \\ C_3, & t_3 \leq t \leq t_4 \\ -60t + C_3, & t_4 \leq t \leq t_5 \\ \vdots & \end{cases} \quad (13)$$

where $t_3 \leq t_1$. Generally, for practical reasons, the decompression stops are taken at 10 foot intervals. In our example, the diver would ascend to 40 feet (he cannot go all the way to 30 feet because at that depth $A > 2P$), and remain there until it is possible to safely ascend to 30 feet. The computations for this situation are done in a manner similar to that given above. The $P(t)$ function given is not optimum, so that it would take longer than 40.5 min to safely surface at the rate given by (1) and (13). Note also that the computations become easier with a piecewise linear function for $P(t)$ in (13).

A MORE COMPLETE MODEL

The procedure described above is generally the one adhered to by scuba divers who plan a sequence of ascents and decompression stops, which enables them to relieve the excess gas pressure in the tissues without causing the bends. However, the theory behind the idea has

gradually evolved, with the aid of a wealth of experimental data gathered over the years by the U.S. Navy, private dive organizations, and other groups.

There are several main features of the current model which differ significantly from the previous one. First, instead of just one type of tissue, we must consider several different types of tissues with varying half-times. In fact, there are as many tissue half-times to consider as there are tissues in the body. For simplicity, we shall consider only six tissue half-times, ranging from 5 to 120 min. Second, it has been found experimentally that the ability of a tissue to ward off the bends by withstanding differences of pressure between itself and the surrounding medium increases with decreasing tissue pressure. For example at four atmospheres of pressure, the ratio of tissue pressure to ambient pressure may be restricted to be no more than 1.9:1 for a certain tissue, whereas at two atmospheres of pressure, that limiting ratio may increase to 2.4:1 for the same tissue. Thus, this limiting ratio (often referred to as the critical pressure ratio) must be considered not as a constant, as in the Haldane theory, but as a decreasing function of tissue pressure.

To facilitate the discussion, we limit ourselves to the consideration of the following tissue half-times: $\tau_1 = 5$ min, $\tau_2 = 10$, $\tau_3 = 20$, $\tau_4 = 40$, $\tau_5 = 80$, $\tau_6 = 120$. Also, we suppose that the functions $f_{\tau_i} = f_{\tau_i}(A)$ are given (perhaps experimentally determined) for each type of tissue (i.e., for each half time, τ_i , $i = 1, 2 \dots 6$).

The reason that we must consider different half-times may not be obvious. Why couldn't we just do the calculations as before for each tissue half-time, and then just choose the proper ascent rate from amongst those six? The problem is that, during descent or ascent, critical pressure ratio curves (functions of time) for the different tissue half-times cross one another. Thus, for part of the ascent, one tissue may be the controlling one, while, for another part of the dive, another tissue may take over control. For example, on a dive from the surface to 198 feet (assume time for descent is short), the 5 min tissue would accumulate enough gas in 5 min to be saturated to a depth of 82.5 feet. On the other hand, after 5 min, the 120 min tissue would have hardly accumulated any excess nitrogen at all. Thus, on ascent, the 120 min tissue could ascend all the way to the surface, but the 5 min tissue would be forced to decompress at a depth of 24.75 feet. At that depth, as well as all intermediate depths, the 5 min tissue would be losing gas, but the 120 min tissue would still be gaining gas. After a while, the 120 min tissue would have accumulated enough gas so that it could not ascend directly to the surface. Then, on ascent, the 5 min tissue would lose its excess gas and be able to return to the surface quickly, whereas the 120 min tissue would require a longer time to surface safely. It has now become the controlling tissue. Control generally shifts from the low half-time tissues to the high half-time tissues during a simple descent-ascent dive.

In our previous example then, the optimum ascent rate must be a composite of different rates. For example, we would begin our ascent at a constant rate. After that, it would be a composite of certain other rates, depending on initial conditions and dive history. Thus, we would have an ascent rate given by (1), with $P(t)$ given by [compare Eq. (8)]

$$P(t) = \begin{cases} -60t + A(0), & 0 \leq t \leq t_1, \\ P_1(t), & t_1 \leq t \leq t_2, \\ \vdots & \\ P_6(t), & t_6 \leq t \leq t_7. \end{cases}$$

Here, t_i is the solution of

$$A(t_i) = f_{\tau_i}(A(t_i))P(t_i), \quad i = 1, 2 \dots 6,$$

$P_i(t)$ is found by solving

$$A(t) = f_{\tau_i}(A(t))P_i(t), \quad t_i \leq t \leq t_{i+1}, \quad i = 1, 2 \dots 6,$$

and t_7 is the total ascent time. $P_i(t)$ may no longer be an exponential function because of the complication introduced by using specified, nonconstant functions f_{τ_i} . In general, we can only hope to solve for $P_i(t)$, $i = 1, 2 \dots 6$, and t_i , $i = 1, 2 \dots 7$ numerically.

SOLUTION OF THE NEW MODEL

The previous examples were used to illustrate the method. Now we present examples for purposes of comparison with the U.S. Navy Table. In the examples, we assume the diver descends to a certain depth at a constant rate (60 fsw/min), remains at the depth for a fixed period of time, then, on ascent, performs a series of ascents (at 60 fsw/min) and decompression stops (for fixed periods of time) at 10 foot intervals, until he is able to surface safely.

We also use the following functions [see inequality (6)] in the calculations:

$$f_{\tau_1}(A) = 132.0/A + 2.8,$$

$$f_{\tau_2}(A) = 99.0/A + 2.1,$$

$$f_{\tau_3}(A) = 79.2/A + 1.68,$$

$$f_{\tau_4}(A) = 64.68/A + 1.372,$$

$$f_{\tau_5}(A) = 55.11/A + 1.169,$$

$$f_{\tau_6}(A) = 53.13/A + 1.127.$$

We now point out some interesting features of the above functions. First, at any given pressure, the function values decrease as we go down the list. In other words the critical pressure ratio for a five minute tissue is larger than that for a 120 min tissue. This indicates that the five minute tissue is less apt to cause the bends than a 120 min tissue, since it can eliminate excess pressure more quickly. Second these functions were chosen (with much trial and error) so as to be able, as much as possible, to duplicate the U.S. Navy Table. This serves two purposes. It allows a comparison with the Navy Table, and it gives the scuba diver more confidence to make a dive that is not listed in the Navy table by following the ascent schedule which can be derived from these functions.

The results of some of the calculations follow. For comparison purposes, we place our results side-by-side with the U.S. Navy Table (Table 1). In the table the depth and time at depth are listed in the two left-hand columns. The table lists the decompression stops required and the length of time at each stop. For example, on a dive to 130 feet for 70 min, the U.S. Navy Table requires the diver to ascend to 30 feet, remain for 16 min, rise to 20 feet, remain for 24 min, rise to 10 feet, remain for 61 min, and then rise to the surface. Our table requires the diver to ascend to 40 feet, remain there for 4 min, rise to 30 feet for 18 min, rise to 20 feet for 28 min, rise to 10 feet for 54 min, and then ascend to the surface. Total ascent time from our table is 3 min more than that from the U.S. Navy Table.

CONCLUSION

The U.S. Navy Dive Table has evolved over the years to its present form in the following way. (1) A mathematical model was developed to produce decompression schedules. (2) The schedules were tested on U.S. Navy divers. (3) The mathematical model was revised, based on the results of the tests, and the updated model was again tested. This procedure continues. See, for example, [5].

This test-and-revise method used to develop the table is unpleasing mathematically. However, the scuba diver has other reasons for not putting complete faith in the table. First,

Table 1. Comparison of U.S. Navy Table and mathematical model ascent schedules

Depth (feet)	Bottom Time (min)	Decompression Stops (feet)										
		U.S. Navy Table					Mathematical Model					
		50	40	30	20	10	60	50	40	30	20	10
40	200					0						0
	210					2						2
	230					7						7
	250					11						11
	270					15						15
	300					19						18
50	100					0						1
	110					3						3
	120					5						5
	140					10						10
	160					21						20
	180					29						28
	200					35						35
	220					40						40
240					47						44	
60	60					0						0
	70					2						2
	80					7						7
	100					14						14
	120					26						25
	140					39						38
	160					48						48
	180					56				2		54
	200				1	69				10		58
70	50					0						3
	60					8						7
	70					14						13
	80					18						18
	90					23						22
	100					33				1		32
	110				2	41				4		38
	120				4	47				6		44
	130				6	52				8		50
	140				8	56				10		54
	150				9	61				16		54
	160				13	72				22		56
170				19	79				27		60	
80	40					0						4
	50					10						9
	60					17						17
	70					23				1		22
	80				2	31				5		27
	90				7	39				9		36
	100				11	46				12		44
	110				13	53				15		50
	120				17	56				19		54
	130				19	63				27		54
	140				26	69				1	33	59
	150				32	77				2	39	63

Table 1. (contd.)

Depth (feet)	Bottom Time (min)	Decompression Stops (feet)											
		U.S. Navy Table					Mathematical Model						
		50	40	30	20	10	60	50	40	30	20	10	
90	30					0						3	
	40					9						8	
	50					18					1	17	
	60					25					4	22	
	70				7	30					9	27	
	80				13	40					14	38	
	90				18	48					19	46	
	100				21	54				2	20	53	
	110				24	61				6	27	54	
	120				32	68				8	34	58	
	130			5	36	74				10	40	64	
	100	25					0						3
30						3						7	
40						15				2	13		
50					2	24				5	21		
60					9	28				11	25		
70					17	39				1	17	37	
80					23	48				3	20	47	
90				3	23	57				8	21	54	
100				7	23	66				12	31	54	
110				10	34	72				15	38	61	
120				12	41	78			1	17	45	67	
110		20					0						3
	25					3						6	
	30					7					1	9	
	40				2	21					5	17	
	50				8	26					9	24	
	60				18	36				3	16	34	
	70			1	23	48				6	20	46	
	80			7	23	57				12	21	54	
	90			12	30	64				17	32	54	
	100			15	37	72			4	17	41	63	
	120	15					0						2
		20					2						4
25						6					1	8	
30						14					3	10	
40					5	25					8	21	
50					15	31				4	13	28	
60				2	22	45				6	20	42	
70				9	23	55			1	13	20	53	
80				15	27	63			3	17	31	54	
90				19	37	74			8	18	41	63	
100				23	45	80			12	21	47	71	
130		10					0						0
	15					1						3	
	20					4					1	6	
	25					10					2	10	
	30				3	18					6	13	
	40				10	25				3	9	23	
	50			3	21	37				7	17	35	
	60			9	23	52			2	12	20	49	
	70			16	24	61			4	18	28	54	
	80		3	19	35	72			11	17	40	61	
	90		8	19	45	80		1	15	21	47	70	

Table 1. (contd.)

Depth (feet)	Bottom Time (min)	Decompression Stops (feet)											
		U.S. Navy Table					Mathematical Model						
		50	40	30	20	10	60	50	40	30	20	10	
140	10					0						1	
	15					2						4	
	20					6				2		7	
	25				2	14				4		11	
	30				5	21				1	7	17	
	40			2	16	26				6	12	24	
	50			6	24	44			2	9	20	41	
	60			16	23	56			5	16	22	54	
	70		4	19	32	68		1	11	17	36	58	
80		10	23	41	79		3	15	19	46	69		
150	5					0						0	
	10					1						1	
	15					3					1	4	
	20				2	7					3	9	
	25				4	17				1	6	13	
	30				8	24				2	9	19	
	40			5	19	33			1	8	15	30	
	50			12	23	51			5	12	20	48	
	60		3	19	26	62		1	9	18	29	54	
70		11	19	39	75		3	16	17	43	65		
80	1	17	19	50	84		10	16	27	47	75		
160	5					0						0	
	10					1						2	
	15				1	4					1	5	
	20				3	11				1	3	10	
	25				7	20				2	7	16	
	30			2	11	25			1	4	9	22	
	40			7	23	39			4	7	19	36	
	50		2	16	23	55		2	6	16	20	53	
60		9	19	33	69		4	13	17	37	58		
170	5					0						0	
	10					2						3	
	15				2	5					2	7	
	20				4	15				1	5	10	
	25			2	7	23				1	2	9	18
	30			4	13	26				1	6	10	24
	40		1	10	23	45		1	6	9	20	42	
	50		5	18	23	61		4	8	17	27	54	
60	2	15	22	37	74	1	6	16	17	44	64		
180	5					0						0	
	10					3					1	3	
	15				3	6					3	8	
	20			1	5	17				2	6	12	
	25			3	10	24			1	4	9	20	
	30			6	17	27			2	8	12	24	
	40		3	14	23	50		2	7	12	20	47	
	50	2	9	19	30	65	1	6	11	17	34	54	
60	5	16	19	44	81	4	10	15	22	46	70		
190	5					0						0	
	10				1	3					1	3	
	15				4	7				1	3	9	
	20			2	6	20			1	2	7	14	
	25			5	11	25			2	5	9	22	
	30		1	8	19	43		1	4	7	15	29	
	40		8	14	23	55		5	6	15	20	52	

the U.S. Navy Table was tested on U.S. Navy divers who tend to be more physically fit than the average scuba diver. For this reason alone, the normal diver should not try to stretch the decompression schedules to their limit. Second, the U.S. Navy Table is incomplete, since many dives which a diver would want to take are not listed. (For example, a dive to 115 feet for 47 min, or a dive to 90 feet for 10 min, followed by an ascent to 50 feet for 15 min.)

The present model is a step toward correcting some of these inadequacies. For instance, the functions f_{τ_i} , $i = 1 \dots 6$, could be decreased for an out-of-shape diver. This would provide an extra measure of protection by increasing the total ascent time. Also, the model could be solved for *any* dive to give an appropriate decompression schedule. In fact, the U.S. Navy has developed an underwater decompression computer which can be worn by the diver. This computer can be programmed, using this model, to monitor the diver's depth and time and to signal the diver when it is unsafe to ascend any further. With such an anaphrotropic decompression schedule at his disposal, the diver can now ascend more quickly to the surface than if he had followed the ascent schedule listed in the U.S. Navy Table. Dieter [4] shows this to be the case, under different hypotheses.

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