Fast & Super-fast Compartments

By Albrecht Salm
The following is an essay on fast and super-fast compartments. So this is not a strict scientific paper, neither in form nor in contents but a couple of preliminary thoughts on the topic, intended to raise awareness or for further discussion.

If you are new to Tech Diving Mag, new to TEC diving or even new to diving, you may enjoy some basic information on deterministic decompression models and algorithms in chapter 2, the „Background“. The seasoned diver may skip this part. Readers not intending to go into the mathematical details may then proceed as well directly to „Take-home Messages“ in chapter 12.

**Chapter 1: Rationale**

During my first course on breath-hold diving some 20 years ago, I stumbled on the inability of standard decompression tables and algorithms to cope with breath-hold diving profiles. My then instructor on this topic, Andy Anlauf, who was at times an elite apea diver, asked me if I could make a decompression table for the record profiles: for example in 2 min down to 130 m and then up to the surface. If you now look at a compartment, say with a halftime ($\tau_{50}$) of 12.5 min (compartment #3 in the standard ZH-L parlance), it will change its initial inertgas load from ca. 0.8 to only 1.1 bar after 1 min @ 90 m. The super-saturation of ca. 0.3 bar is not enough to yield any basic decompression; even on return to the surface it is still taking up inertgas and the super-saturation is raised to ca. 0.5 bar, still not sufficient for a substantial decompression time. The other compartments from # 8 on will not even take note on this pressure excursion.

Further on, there is a phenomenon called „Taravana“: these are the many anecdotal reports on unexplained DCS cases during breath-hold dives, especially for commercial indigenous sea harvesters.

As well Paulev (cf. chapter 11) observed cases of DCS type II during breath-hold submarine escape training; Schaefer (cf. Chapter 4) observed N₂ bubbles in blood samples from breath-hold divers, quickly disappearing after 10 sec.

In the TEC community there is since long a sometimes overheated discussion around the effectiveness of short, 1 to 2 min, deep stops during decompression from mixed gas dives.

The time domain of all these phenomenon is in the sub-5 min region. Basically a phenomenological description needs thus an exponential halftime ($\tau_{50}$) in the order of a fraction of the maximal time-frame. Thus approximately 5 min divided by 6 halftimes would allow for a clean description to cope mathematically with the quick pressure changes: 6 halftimes being the rule-of-thumb for complete saturation or desaturation of any compartment (at constant pressure). We end up thus with $\tau_{50}$ of approx. 60 sec.

After a snappy introduction to decompression models and algorithms in the next chapter, there will be a short and limited literature overview which reveals if and how other selected researchers have been dealing with the spectrum of used halftimes.

**Chapter 2: Background: What is a compartment, anyway??**

The following is a boldfaced copy from a book of Carl Edmonds, another chap of mine (Ref.: Edmonds, Carl. *Diving and Subaquatic Medicine, Fifth Edition*. CRC Press, 20150713. VitalBook file), the graphs used here have been drawed originally by Dr. David Doolite, working now for the Naval Experimental Diving Unit (NEDU) of the United States Navy (USN):
lines in the graph and with the rule-of-thumb cited above you can derive the halftimes of the compartments. For example $P_1$ reaches its 50% saturation after 5 min, so after $6 \times 5 = 30$ min it is supposed to be saturated; $P_2$ after $6 \times 10$ min.

(With a friendly permission by Carl Edmonds and David Doolite)

The box depicted above is a model for the limited volume of some region in a mammalian body: one compartment is showed here. It is a model for a well-stirred tissue (thus the symbol with the little mixer) with a defined, perfusion-limited blood supply: the arrows from left, the arterial part to the right, the venous part.

Then we will look at a dive scenario with more compartments: we see the nitrogen uptake in five hypothetical perfusion-limited tissue compartments during a dive to 30 metres (4 ATA) using air. $P_{amb}$ is the ambient pressure in atmospheres (atm). The inspired pressure of nitrogen and the alveolar pressure of nitrogen rise to ~3.1 atm (not depicted in the figure), and the arterial pressure of nitrogen ($P_{aN_2}$) immediately equilibrates. The tissue pressures of nitrogen are slower to equilibrate, due to the final capacities of the blood, lung and circulation carrying the inert gases. Only tissues 1 and 2 approaching saturation within the duration of the exposure depicted. From the

(With a friendly permission by David Doolite)

The lines of saturation follow an exponential curve, typical for many natural phenomena, the math behind a simple linear differential equation is already described elsewhere, for example here: https://www.divetable.info/theory.htm.
In this model we have $P_1$ to $P_5$ in a parallel circuit (cf. graph below, the lower part), other models with a serial circuit are possible as well. The most prominent decompression models like the ones from Haldane, Workman (USN tables), Schreiner and Bühlmann (ZH-L) are using the parallel perfused setup. The serial circuit showed below (upper part of the graph) is used by Kidd, Stubbs, Nishi et al for the DCIEM tables and Canadian military and commercial procedures. We see 4 compartments designated # 1 to # 4, with halftime $\tau_{1/2} :$ HT 1 to HT 4. In the serial setup there need not to be different values.

**Serial versus parallel coupling of compartments**

A completely other game is a „statistical“ decompression model: there the outcome of thousands of dives is analysed after surfacing. The outcomes (DCS: YES or NO) being fitted to a model and then a decompression table with a defined probability of getting DCS is derived.

**Physiologic definition of the compartment halftime**
As was described earlier, the halftime ($\tau_{1/2}$) are related to the change in the moved blood volume, i.e. the volume per time (ml per min) per ml of compartment volume; thus the physiologic definition looks like that:

$$\tau_{1/2} = 0.693 \times \frac{\alpha_t}{\alpha_b} \times \left(\frac{dQ}{dt}\right)$$  \hspace{1cm} (0)

where:

- $\alpha_t$: solubility of the inert gas per compartment (tissue = ti), ml_{(S)gas} * ml_{ti}^{-1} * (100 kPa)^{-1}
- $\alpha_b$: solubility of the inert gas in blood (blood = bi), ml_{(S)gas} * ml_{blood}^{-1} * (100 kPa)^{-1}
- $dQ/dt$: perfusion rate, ml_{blood} * ml_{ti}^{-1} * min^{-1}

The ratio of the solubilities blood / tissue ($\alpha_b / \alpha_t$) has a well-known name: the „partition coefficient“; it could be looked up in tables (cf. the remarks on PBPK in chapter 8). If you do not have the partition coefficient of your compartment in question and you do not have a clue about its perfusion rate, you collapse everything into a single value. This approach leads directly to the pragmatic Schreiner matrix (cf. chapter 5).

**A compartment as a „low pass“!**
The exponential functions to describe the on/off gasing of the compartments are nearly the same for an electronic circuit, consisting of a capacitor and a resistor. It is used for example to rectify the
current from AC to DC: the high frequency parts of the AC are filtered, allowing only the lower frequencies to pass the electronic circuit; thus the name „low pass“.

Now, if you have a part of your dive profile with a „high frequency“ behavior, i.e. noticeable changes of the diving depth versus short times as in yo-yo diving, the decompression algorithm is „blind“ for it: the dive computer may log the depth changes over time but the slower compartments will never notice it. (Ref.: Hahn MH (1989): Reponses of decompression computers, tables and models to „yo-yo“ diving, Undersea Biomed Res 16 (Suppl.): 26.)

**Chapter 3: Experiment with goats: Haldane**

The set of halftimes for his 5 compartments was generated by just doubling the 5 min halftime 3 times, with the longest halftime being 75 min due to a hypothetical saturation of nitrogen uptake at around 5 to 7.5 h (pages 349 and 350) for the goats he used for his experiments: 5, 10, 20, 40 and 75 min. Then there could be as well a compartment with a halftime of 2.5 or 1.25 min. On page 348 he gave a hint to a faster saturation process within max. 10 min which would yield a halftime of: 10 min / 6 → ca. 1.6 min.

We could easily exploit this with his rule for safe ascent, the famous „2:1“ rule to generate a „new“ haldanian-type decompression table, but with deep stops! These stops being noticeably deeper than in the original tables, in the 1 min region and not altering the shallow stops by much [an easy procedure on how to do that and an appreciation of the work of Haldane and his colleagues you will find in this magazine, cf. Tech Diving Mag, Issue 25 (December 2016), on pages 13 – 20].

**Chapter 4: Submarine escape: Schaefer**
In his 1955 contribution to the first Underwater Physiology Symposion, he presented his paper titled: The role of carbon dioxide in the physiology of human diving, Schaefer describes on page 135 that during breath-hold dives in the 90 feet submarine escape training tank there have been bubbles observed in alveolar and venous blood samples which have been attributed to N₂ and not to CO₂. The blood samples were drawn from the divers immediately on surfacing after a breath-hold dive. The foam due to these bubbles may have been disappearing 10 sec after surfacing or 40 sec after start of ascent, the duration of these dives being ca. 1 to max. 2 min. An allowable supersaturation ratio of 3:1 seems to be exceeded.

This in turn would imply a de-saturation with a halftime of approx. 10 + 40 / 6 (ca. 10 sec) and a saturation process with a halftime from 1/6 min up to 2/6 min.

**Chapter 5: The pragmatic Schreiner matrix**
In this contribution to the fourth Symposion in 1971, Schreiner and Kelley presented their paper titled: A pragmatic view of decompression.

As we can see in the following page, the pragmatic 4 by 4 matrix of the 16 compartments, compartment # 0 is never used. That is: we (*) could easily extract a super-fast compartment with a halftime of 2.5 or 1.25 min by exploiting his scheme on page 210 with dQ/dt * R = 0.2772 min⁻¹ resp. 0.5544 (fat fraction X = 0.0)
Fig. 2. Derivation of inert gas exchange compartments by the arbitrary pairing of four specific rates of tissue perfusion and four levels of tissue fat fraction. The resulting compartments are numbered 0 to 15 as shown.

* min⁻¹ in solving Eq. (13), one obtains a total of 16 different values of k representing 16 inert gas exchange units or compartments. These entities are not necessarily identifiable anatomical substructures of the body but rather represent assemblages of those regions within the human body that happen to be characterized by one and the same specific time constant of inert gas transport. These 16 inert gas exchange compartments (numbered 0 to 15 for ease of reference) are shown schematically in Fig. 2. It is immediately clear that any other arbitrary array of Q/R and z may be employed to derive gas exchange compartments as long as representative and minimal rates of the specific rate of tissue perfusion and extreme values of fat fraction are included.

Chapter 6: United States Navy method: Workman

Here we have compartment halftimes for N₂ from 5 to 240 min (p. 5) and the corresponding allowed inert gas super-saturations, called M-Values. The M-value follows a simple linear relationship, based on empirical dive data (Eq. 1):

$$M = M_0 + \Delta M \cdot d$$  \hspace{1cm} (1)

where $M_0$ is the maximum inert gas partial pressure in the compartment for surfacing and $\Delta M$ is the change with the diving depth (in feet). By fitting separately the $\Delta M$ (Delta M) and $M_0$ over the halftimes we (*) could as well extract faster compartments and the corresponding allowed super-saturations.

Fit for $M_0$
Our generator function yields with a correlation coefficient of nearly 1, for example for the halftimes 1.25, 2 and 2.5 min these values for $M_0$ are 156, 134 and 126 fsw respectively.

Fit for $\Delta M$
The above generator polynom gives here, as well with a very high correlation coefficient for the same chosen halftimes of 1.25, 2 and 2.5 min these $\Delta M$ values are 37.5, 8.4 and 4.5 respectively.

Chapter 7: Swiss altitude diving: Bühlmann
Here we have already a simple relationship between the halftime \( \tau_{1/2} \) of a compartment and the allowed super-saturation for \( N_2 \). If we combine the two empirical relationships for the coefficients \( a \) & \( b \) from p. 129 (Eq. 2) with the linear equation for the tolerated ambient pressure (p. 117) (Eq. 3) into one:

\[
\begin{align*}
(2) & \\
a &= 2.0 \text{ bar} \times (\tau_{1/2} N_2 [\text{min}])^{-1/3} \\
b &= 1.005 - 1 \times (\tau_{1/2} N_2 [\text{min}])^{-1/2}
\end{align*}
\]

\[P_{\text{compartment}} = \left( \frac{P_{\text{ambient, tolerated}}}{b} \right) + a \] (3)

This yields the following generator function (Eq. 4) by setting the tolerated ambient pressure to 1 bar (for a direct ascent to the surface for breath-hold diving or submarine escape training):

\[P_{\text{compartment}} = \left(1 \text{ bar} / (1.005 - \tau^{-1/2})\right) + (2 \text{ bar} \times \tau^{-1/3}) \] (4)

Thus we could extract here as well faster compartments and the corresponding compartment overpressures. Here around a halftime of \( \tau_{1/2} = 1.005 \text{ min} \) is a divergence in (Eq. 4) and thus this is the smallest allowed value.

Our chosen halftimes of 1.25, 2 and 2.5 min are yielding the compartment overpressures of ca. 11, 4.95 and 4.1 bar respectively. These we could compare directly with the \( M_0 \)-values from the Workman set above, i.e. for \( d = 0 \text{ fsw} \) in (Eq. 1): 4.8, 4 and 3.9 bar respectively.

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**Chapter 8: PBPK: Mapleson, Nishi, Flook et al.**

One of the first PBPK (Physiologically Based Pharmacokinetic) models solved via a simulation with an electric analog circuit was the one from Mapleson, intended to simulate the uptake of inhaled narcotic gases like halothane in the human body:


Others, like: Morales, M.F. and R.E. Smith, 1944, 1945 and1948 in: Bulletin of Mathematical Biophysics, have not been successfully solved at that time due to a lack of fast-enough hardware.

Since then the PBPKs are used to simulate as well drugs and other environmental influences on the human body: by the same token we could designate the Haldane model as one of the first PBPKs.


Here we find as well super-fast compartments, i.e. # 1 and 2 in the following table:

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The perfusion rates vary not only with a factor of 250 from ca. 20 (bones) to 5000, but as well over time course and authors. This variance should be reflected as well in the spectrum of used halftimes for a decompression algorithm. As well there are data for just 14 compartments, meaning that using a lot more, as some of dive computers do, would probably not give any further clues. The only argument of using more being philosophically, that “Nature does not make leaps” (Gottfried Wilhelm Leibniz: La nature ne fait jamais de sauts).

**Chapter 9: Mixing two models: Egi & Gürmen**


The authors were considering the Workman and as well the Bühllmann framework. But instead of fitting each set of M-values to the appropriate halftimes within the corresponding framework they fitted all M-values to all halftimes in a hybrid manner and such combining the Workman and Bühllmann values. The result is a smoothed M versus halftime function with high correlation coefficients. The plot of ln(M) versus ln(halftime) yields a straight line (Fig. 7 on page 149):

<table>
<thead>
<tr>
<th>Compartment</th>
<th>Tissues</th>
<th>Time constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Adrenals, kidneys, thyroid</td>
<td>0.86</td>
</tr>
<tr>
<td>2</td>
<td>Heart, brain grey matter</td>
<td>1.87</td>
</tr>
<tr>
<td>3</td>
<td>Liver plus portal system, other small glands and organs</td>
<td>3.07</td>
</tr>
<tr>
<td>4</td>
<td>Brain white matter</td>
<td>5.31</td>
</tr>
<tr>
<td>5</td>
<td>Red marrow</td>
<td>12.25</td>
</tr>
<tr>
<td>6</td>
<td>Muscle and skin</td>
<td>50.62</td>
</tr>
<tr>
<td>7</td>
<td>Nonfat subcutaneous</td>
<td>69.14</td>
</tr>
<tr>
<td>8</td>
<td>Fatty marrow and fat</td>
<td>211.3</td>
</tr>
</tbody>
</table>

*Reference values for resting blood flow to organs of man: R Williams* and R W Leggett; Metabolism and Dosimetry Research Group, Health and Safety Research Division, Oak Ridge; National Laboratory, Oak Ridge, Tennessee 37831-6383, USA, 21 February 1989. On page 188 we have a compilation of the relevant perfusion values:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Adipose tissue</td>
<td>20</td>
<td>–</td>
<td>24</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Adrenals</td>
<td>5000</td>
<td>5100</td>
<td>540</td>
<td>540</td>
<td>540</td>
<td>540</td>
</tr>
<tr>
<td>Bone</td>
<td>0</td>
<td>120</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Brain</td>
<td>510</td>
<td>650</td>
<td>530</td>
<td>540</td>
<td>540</td>
<td>540</td>
</tr>
<tr>
<td>Lung tissue</td>
<td>–</td>
<td>1000</td>
<td>1000</td>
<td>700</td>
<td>840</td>
<td>610</td>
</tr>
<tr>
<td>Heart tissue</td>
<td>800</td>
<td>1000</td>
<td>810</td>
<td>700</td>
<td>840</td>
<td>610</td>
</tr>
<tr>
<td>Intestines</td>
<td>–</td>
<td>700</td>
<td>390</td>
<td>540</td>
<td>700</td>
<td>700</td>
</tr>
<tr>
<td>Kidneys</td>
<td>4100</td>
<td>1500</td>
<td>4090</td>
<td>4300</td>
<td>4220</td>
<td>3600</td>
</tr>
<tr>
<td>Liver (total)</td>
<td>410</td>
<td>1500</td>
<td>810</td>
<td>540</td>
<td>580</td>
<td>750</td>
</tr>
<tr>
<td>Red marrow</td>
<td>90</td>
<td>30</td>
<td>230</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Skeletal muscle</td>
<td>20-50</td>
<td>20</td>
<td>21</td>
<td>27</td>
<td>27</td>
<td>26</td>
</tr>
<tr>
<td>Skin</td>
<td>20-50</td>
<td>30</td>
<td>57</td>
<td>–</td>
<td>130</td>
<td>120</td>
</tr>
<tr>
<td>Spleen</td>
<td>–</td>
<td>400</td>
<td>390</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Thyroid</td>
<td>4000</td>
<td>5600</td>
<td>5080</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
If we exploit this function with $x = 0.25$ (i.e.: halftime = 1.28 min) the results are $M_0 = 117$ fsw; with $x = 0.1$ (halftime = 1.1 min) yields $M_0 = 126$ fsw.

**Chapter 10: Breath-hold and DCS Type II: Goldman et al.**

(Ref.: Decompression sickness in breath-hold diving, and its probable connection to the growth and dissolution of small arterial gas emboli; Saul Goldman, J.M.Solano-Altamirano, Mathematical Biosciences 262 (2015): 1–9.)

In this paper we find a super-fast compartment (brain) with the halftime of 72 sec.

![Graph showing correlation between ln(M) and ln(T1/2) for hybrid data set (Workman and Bühmann) at sea level. Table 4 lists the slope of the line and y-intercepts of different data set combinations.](image)

**Chapter 11: A Fit to the Paulev data**

To be completely honest with my sources, I recieved the Paulev papers from Karl Huggins, with whom I started to discuss this topic around the turn of the millenium. Karl created his version of a USN deco table („HUGI table“) as well he was fundamental for the ORCA EDGE dive computer in the 80s (The ORCA EDGE being one of the first diver carried computers not only interpolating stored table values but instead using a full-blown decompression model). Paulev, as described in the „Rationale“, observed on himself a case of neurological DCS during submarine escape training (ref. 1) which has been treated successfully in a deco chamber. Subsequently he made measurements of exhaled gases during breath-hold diving (refs. 2 and 3):


From this published curve (Fig. 1 on page 715 in paper 3; as well the Fig. 3 on page 438 in the paper 2), we (*) extracted graphically the raw data in order to simulate the N₂ uptake of one super-fast compartment. A fit to a mono-exponential saturation function like:

$$Y = 1 - a \cdot \exp(-b \cdot X) \quad (5)$$

Where \( Y = N₂ \) Saturation, alveolar [%] and \( X = \) dive time [seconds] yields the following:

\( a = 0.24 \)
\( b = 0.01 \)

with a relatively high correlation coefficient around 0.97; the mathematical details are too specific for an essay like this. But anyway there is:

**Error propagation**

We end at an error of approx. +/- 12 % of the fitted values due to uncertainties of the published graphical data, which is not available in digital form.

**Halftime of the super-fast compartment**

Thus the halftime is, by definition, \( \tau_\% = \ln 2 / b = \text{ca. 70 sec} +/- 12 \text{ to 15 %} \), with a stunning coincidence with Saul's value (chapter 10). This one would give, in return to the \( a \) and \( b \) coefficients of Eq. (2), a maximal inert gas partial pressure (4) in this „fast compartment“ of 8 up to ca. 20 bar within the Bühmann framework. One could question the sheer size of this value derived from the model directly, but presently there are not enough data at hand. On the other hand, there are no arguments for not keeping the maximal tolerated overpressure from the fastest compartment as well for the super-fast compartments. Thus we could designate the ca. 3.5 bar overpressure from the traditional 2.5 to 5 min compartment to the faster ones.
Chapter 12: Take-home messages
A compartment is not a single physiological site in the body, instead, it is a group of various tissues, sharing some common properties, like the perfusion rate, which is basically the invers of the halftime used in the exponential curves.

If you use more compartments, say in your dive computer or a decompression model, you do not get closer to the truth, instead you just get closer to the data points at hand.

For fast processes, like yo-yo diving or breath-hold profiles, the usually used half-times are by far too slow, i.e.: the dive computer (resp. the decompression model) acts like a „low pass“.

To simulate processes like that, you need faster and/or super-fast compartments, namely in the sub-min region, like a halftime $\tau_{\frac{1}{2}}$ from 30 sec to 1.5 min.

(*): SubMarineConsulting: www.SMC-de.com